The Prometheus Orthonormal Set for Wideband CDMA

Hakan Deliç[1], James S. Byrnes[2] and Gerald Ostheimer[2]
[1]BUSIM Lab., Department of Electrical and Electronics Engineering, Boğaziçi University
Bebek 34342 Istanbul, Turkey
delic@boun.edu.tr.
[2]Prometheus Inc, 21 Arnold Avenue, Newport, Rhode Island 02840, USA
{jim,gerald}@prometheus-inc.com

Abstract— **The Prometheus orthonormal set (PONS) represents a Hadamard-type sequence construction with advantageous autocorrelation, spectrum and peak-to-average power ratio properties. A PONS-based orthogonal variable spreading factor (OVSF) sequence set is proposed for channelization in wideband code-division multiple access (WCDMA) applications. A duocode version is shown to yield the same code capacity as Walsh-based OVSF. The duocode OVSF-PONS channelization offers a strong alternative to Walsh-based constructions in next generation wireless communications, due its superior sequence properties which can be implemented at the expense of slightly increased complexity.**

Keywords—**Wideband CDMA, orthogonal variable spreading factor, Walsh sequences, Prometheus orthonormal set, Golay complementary sequences**

I. Introduction

The third-generation Universal Mobile Telecommunication Systems (UMTS), which is based on the wideband CDMA (WCDMA) concept, envisions data transfers at a nominal rate of 384 kbps, aside from the traditional voice services. All users spread their data to a 5 MHz bandwidth by replacing each information bit with a unique code, which is a sequence of shorter duration *chips*, and simultaneously transmit over the same wideband channel. The codes are designed to be orthogonal so that the corresponding despreading at the receiver recovers the desired message, while canceling other users' signals and spreading any narrowband interference to a low amplitude.

The deployment of WCDMA in third-generation UMTS relies on orthogonal variable spreading factor (OVSF) sequences for channelization in both uplink and downlink [5], [7]. High-rate users are assigned codes with low spreading factors. Starting with the 2×2 Hadamard matrix, a tree-structured method generates 2^m spreading codes of length-2^m chips from the 2^{m-1} parent code of length-2^{m-1} by concatenating each row of the parent code matrix with itself and its complement [1], [7]. In particular, the length-2^m sequence set $\mathcal{W}_n$ is generated from the length-2^{m-1} one in the following manner:

$$\mathcal{W}_n = \begin{bmatrix} c_{2^m,1} \\ c_{2^m,2} \\ c_{2^m,3} \\ c_{2^m,4} \\ \vdots \\ c_{2^m,2^m-1} \\ c_{2^m,2^m} \end{bmatrix} = \begin{bmatrix} c_{2^{m-1},1} & c_{2^{m-1},1} \\ c_{2^{m-1},1} & -c_{2^{m-1},1} \\ c_{2^{m-1},2} & c_{2^{m-1},2} \\ c_{2^{m-1},2} & -c_{2^{m-1},2} \\ \vdots & \vdots \\ c_{2^{m-1},2^{m-1}} & c_{2^{m-1},2^{m-1}} \\ c_{2^{m-1},2^{m-1}} & -c_{2^{m-1},2^{m-1}} \end{bmatrix}, \quad (1)$$

where $\{c_{2^m,i}\}_{i=1}^{2^m}$ is the Walsh set of sequences of length 2^m.

The UMTS standard utilizes OVSF codes with $m = 2, 3, \ldots, 8$ for the uplink and $m = 2, 3, \ldots, 9$ for the downlink transmissions. Despreading through multiplication by the appropriate code at the receiver recovers the desired data, due to the orthogonality among the OVSF codes, provided that chip synchronization is maintained [7].

The performance of next generation cellular systems, OVSF- or multicode-based, will depend on the correlation properties of the code-set that is employed. Due to multipath effects, non-negligible timing offsets causing asynchronism and hence non-zero cross-correlations will occur at the receiver. While the resulting multiple access interference (MAI) and intersymbol interference can be combatted by equipping the receiver with complex equalization and detection devices, efforts should be undertaken to design OVSF sequences with more suitable correlation functions.

The OVSF code construction proposed by [1] (henceforth called OVSF-Walsh) inherits the shortcomings of Walsh sequences. In particular, the autocorrelation function is nonzero around the maximum, leading to synchronization difficulties; energy spreading is inefficient with a concentrated mainlobe in the frequency spectrum; and the peak-to-average power ratio is high.

In this paper, it is shown that the Prometheus orthonormal set (PONS) development inherently possesses an OVSF structure, making it suitable for WCDMA with the promise of better performance. Because it is possible to obtain PONS from the Walsh-Hadamard matrices with a straightforward transformation, existing hardware and software can accommodate PONS with minimal additional cost. The advantages of PONS-based OVSF codes come at the expense of a minor reduction in the number of available codewords in certain cases, which can be overcome by fusing the OVSF and multicode CDMA concepts, as will be explained below.

II. THE PROMETHEUS ORTHONORMAL SET

The original development of PONS is based on the Shapiro polynomials, which have coefficients ± 1 [16]. To prove that a certain global uncertainty bound is satisfied, Byrnes expanded the Shapiro polynomials via the concatenation rule depicted below [3]. Working with the sequences formed by the polynomial coefficients, the PONS matrices are as follows. Starting with

$$\mathcal{P}_1 = \begin{bmatrix} P_{1,1} \\ Q_{1,1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$$

Concatenation leads to

$$\mathcal{P}_2 = \begin{bmatrix} P_{2,1} \\ Q_{2,1} \\ P_{2,2} \\ Q_{2,2} \end{bmatrix} = \begin{bmatrix} P_{1,1} & Q_{1,1} \\ P_{1,1} & -Q_{1,1} \\ Q_{1,1} & P_{1,1} \\ -Q_{1,1} & P_{1,1} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 \end{bmatrix},$$

and letting

$$\mathcal{P}_{m-1} = \begin{bmatrix} P_{m-1,1} \\ Q_{m-1,1} \\ \vdots \\ P_{m-1,2^{m-2}} \\ Q_{m-1,2^{m-2}} \end{bmatrix},$$

which is of dimension $2^{m-1} \times 2^{m-1}$ with each row being one of the 2^{m-1} PONS sequences, the $2^m \times 2^m$ PONS matrix is obtained as

$$\mathcal{P}_m = \begin{bmatrix} P_{m,1} \\ Q_{m,1} \\ P_{m,2} \\ Q_{m,2} \\ \vdots \\ P_{m,2^{m-1}-1} \\ Q_{m,2^{m-1}-1} \\ P_{m,2^{m-1}} \\ Q_{m,2^{m-1}} \end{bmatrix} = \begin{bmatrix} P_{m-1,1} & Q_{m-1,1} \\ P_{m-1,1} & -Q_{m-1,1} \\ Q_{m-1,1} & P_{m-1,1} \\ -Q_{m-1,1} & P_{m-1,1} \\ \vdots & \vdots \\ P_{m-1,2^{m-2}} & Q_{m-1,2^{m-2}} \\ P_{m-1,2^{m-2}} & -Q_{m-1,2^{m-2}} \\ Q_{m-1,2^{m-2}} & P_{m-1,2^{m-2}} \\ -Q_{m-1,2^{m-2}} & P_{m-1,2^{m-2}} \end{bmatrix}. \quad (2)$$

In addition to leading directly to an "energy spreading" basis for $L_2(R)$, PONS functions possess the following qualities which are absent in Walsh functions [3], [4].

1. Global uncertainty bounds are satisfied.
2. Crest factor of $\sqrt{2}$ is attained for all m (see also [2]).
3. $P_{k,i}$ and $Q_{k,i}$ satisfy the quadrature mirror filter (QMF) definition and are complementary pairs for $k = 1, 2, \ldots$ [9].

The significance of the advantages that a PONS-based OVSF code-set offers is better understood when the autocorrelation and spectral properties of the sequences are investigated. In particular, PONS has (i) an impulse-like autocorrelation making it much more amenable for synchronization (see Fig. 1); (ii) uniformly spread frequency spectrum making it more resistant to frequency-selective fading and interference (see Fig. 2 with a comparison to Walsh spreading); (iii) peak-to-average power ratio (PAPR) of 2 making it energy- and hence battery-efficient. Spreading with PONS rather than Walsh sequences yields signals requiring lower short-term peak power to maintain a specified average transmission power. This is advantageous in view of the power control issues involved in CDMA systems to address the near-far effect.

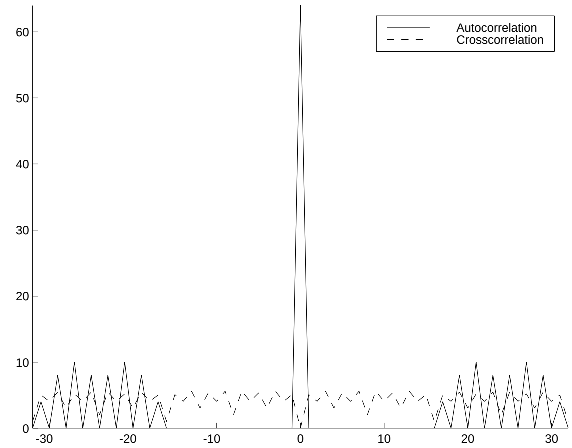

Fig. 1. Magnitude plots of PONS correlation averages for $m = 6$. The x-axis shows the amount of shift.

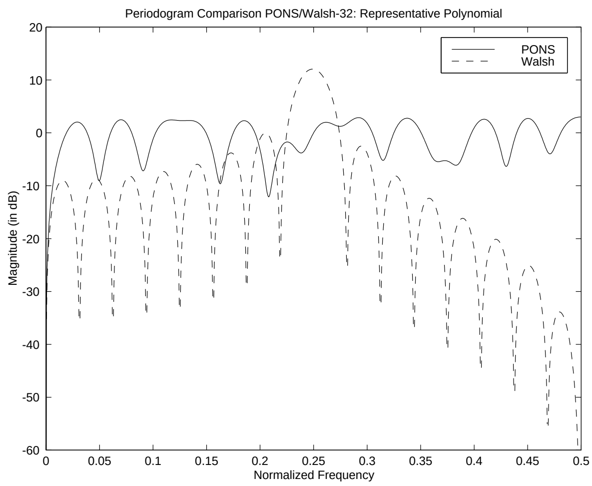

Fig. 2. The periodograms of PONS and Walsh representative polynomials for $m = 5$.

Note that the good behavior of PONS is inherited from the structure of Shapiro polynomials and the associated sequences, recognized as the Golay complementary sequences in the engineering community [9]. Indeed, the low PAPR feature of complementary sequences has inspired researchers to employ them in orthogonal frequency division multiplexing (OFDM) systems [6], [10], [13].

Within the framework of multicarrier CDMA (MC-CDMA), Popović has demonstrated that the Walsh sequences have the worst performance in terms of crest factor, dynamic range of the complex signal envelope, and the average bit error probability among various spreading sequences [14]. In contrast, complementary sequences display the lowest average bit error probability.

Based on the discussion so far, PONS is a powerful candidate for OVSF coding in next generation mobile communications due to its low PAPR; uniform energy spreading in the frequency domain; potential for the best average bit error probability performance; and the structure that naturally lends itself to OVSF construction.

III. OVSF-PONS

Let layer m correspond to the OVSF codes of length 2^m. At layer m, there are 2^m mutually orthonormal PONS codes. Higher layer codes are assigned to low-rate users. The construction of $\mathcal{P}_m$ from the parent rows of $\mathcal{P}_{m-1}$ presents itself naturally as a foundation for OVSF code design.

Note that while all $\mathcal{P}_k$, $k = 1, 2, \ldots$, are Hadamard, the simultaneous utilization of any code at layer m, and either of its parents at layer $m-1$ (through repetition) for channelization will result in loss of orthogonalization between 2^{m-1} chips. Hence, the whole bit of layer m and half the bit of layer $m-1$ will collide with maximum interference, if perfect synchronization is assumed at the receiver.

Define the *admissible OVSF set* to be the collection of codes that are mutually orthogonal. Orthogonality between codes that belong to different layers is established through repetition of the shorter code. Considering equation (2) at layer m, define further the kth quadruplet as

$$\begin{aligned} \mathcal{C}_m(k) \quad = \quad & \{\{P_{m-1,k}, Q_{m-1,k}\}, \{P_{m-1,k}, -Q_{m-1,k}\}, \\ & \{Q_{m-1,k}, P_{m-1,k}\}, \{-Q_{m-1,k}, P_{m-1,k}\}\}, \end{aligned}$$

$k = 1, 2, \ldots, 2^{m-2}$. If any code in $\mathcal{C}_m(k)$ is assigned to a user, the parent codes $P_{m-1,k}$ and $Q_{m-1,k}$ of layer $m-1$ can no longer be employed for spreading due to lack of orthogonality with $\mathcal{C}_m(k)$. Thus, the use of any code at layer m results in the exclusion of two codes from the admissible OVSF set at the higher-rate layer $m-1$. Likewise, if either $P_{m-1,k}$ or $Q_{m-1,k}$ is occupied, the entire quadruplet $\mathcal{C}_m(k)$ has to be dropped from the admissible OVSF code-set.

In contrast, the OVSF-Walsh construction is such that when a code from layer m is assigned, only one code from layer $m-1$ is dropped out of the admissible OVSF set, because each sequence at the former layer is generated from a single parent. Similarly, the use of a code at layer $m-1$ implies the loss of two codes at layer m.

Based on the observations above, one can see that when a single parent code is in use, the OVSF-PONS construction produces two fewer admissible sequences at the next layer down. This fact is true when any odd number of parent codes are employed as long as complementary sequences are exhausted first in code assignment. That is, the code allocation strategy should be such that if $P_{m-1,k}$ is already in use, the next assignment should be $Q_{m-1,k}$ so that premature code blocking is avoided. Moreover, for even number layer-k code allocations, the code capacity of OVSF-PONS is the same as OVSF-Walsh, again provided that complementary pairs are given out together.

Generalizing from the preceding discussion, the number of inadmissible OVSF-PONS sequences at layer j per complementary pair of occupied codes (regardless of whether only one or both are in use) at layer i, $i < j$, is 2^{j-i+1}. On the other hand, the inadmissible OVSF-Walsh sequences at layer j amount to 2^{j-i} per allocated code at layer $i < j$. Depending on the depth of the low-rate codes and whether an odd or even number of high-rate codes are in demand, the capacity difference between OVSF-Walsh and OVSF-PONS may be significant. Next, we propose the *duocode* OVSF-PONS scheme, which overcomes this problem.

IV. DUOCODE OVSF-PONS

Multicode CDMA satisfies the variable bit rate demands by allocating multiple spreading sequences to each user [11]. The data are converted from high-rate serial to two low-rate parallel bit streams, following channel coding and interleaving. Each parallel bit subsequence is spread by a distinct code. The higher the rate or priority required by a transmission, the more codes are assigned to the corresponding unit. Due to its strong suitability for accommodating multimedia communications, multicode WCDMA is under consideration for next generation wireless networking [8]. While there are no conclusive reports, the general view favors OVSF-based multirate CDMA over multicode CDMA owing to the (i) lower complexity, (ii) better PAPR performance, and (iii) ease of synchronization in channels with wide coherence bandwidth [12] of the former.

Considering the OVSF-PONS framework described in the previous section, suppose that a user requires a code from layer $m-1$. If either $P_{m-1,k}$ or $Q_{m-1,k}$, $k = 1, 2, \ldots, 2^{m-2}$, is occupied, then all codes in $\mathcal{C}_m(k)$ and their descendants become inadmissible. On the other hand, if the user's data are spread with any of the two layer-m codes in $\mathcal{C}_m(k)$ instead, then the remaining two (or their descendants) can still be employed by others. Therefore, the OVSF-PONS scheme is modified such that any user that requires a layer-$m-1$ code may be given two layer-m PONS codes depending on the code capacity constraints at the time of signalization. The integrated duocode-OVSF structure (henceforth called the duocode OVSF-PONS) has the same code capacity as the OVSF-Walsh construction. However, it should be stressed that the quality of service (QoS) satisfaction as far as the bit rate requirements are concerned is handled by OVSF coding. The duocode mechanism introduced here is just a part of the OVSF-PONS implementation, and does not in itself serve QoS control purposes (unlike, e.g., the multicode scheme in [17]).

By spreading the two parallel data streams with longer codes, the processing gain is doubled. Hence, duocode OVSF-PONS provides a higher degree of multipath and multiple access interference (MAI) resistance, in addition to the statistical qualities that are inherent in PONS sequences. It is certainly possible to extend duocode OVSF-PONS to a multicode OVSF-PONS structure by replacing a layer-$m-1$ code with four codes from layer $m+1$. (In fact, the duocode OVSF-PONS multiuser spread spectrum system becomes a PONS-based multicode CDMA in the limit.)

V. CONCLUSIONS

The duocode extension of OVSF-PONS does not compromise the code capacity, while promising superior performance and power efficiency because of the autocorrelation and spectral characteristics of the PONS sequences. The performance gains offered by the duocode OVSF-PONS channelization come at the expense of added complexity due to the serial-to-parallel conversion (and vice versa) and multiple simultaneous spreading, as well as slightly increased PAPR when compared to basic PONS coding.

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